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Approximate Four Jet Cross Sections [1]

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Abstract

Approximate matrix elements squared are given for all parton processes involving a quark-antiquark pair plus four gluons. Detailed comparisons are made between these results and the exact matrix elements for the subprocesses involving a quark-antiquark pair and four gluons in four jet production at hadron colliders. Together with Maxwell's approximate result for six gluon processes an excellent agreement is found for the shape and total cross section of four jet production.

1 Introduction

Recently there has been a great deal of interest in four jet events at hadron colliders by both experimentalists[1] and theorists[2]. For four jet production all QCD matrix elements exist in the literature[3] but intense computer usage is required to calculate each partonic cross-section. Sometime ago, Maxwell[4] gave a systematic procedure to approximate multi-gluon cross-sections and he also suggested the use of the effective-structure-function approximation[5] to describe processes involving quarks as well. However this approximation for the fermionic cross-sections is known to become progressively worse with an increasing number of partons.

Here we generalize the multi-gluon approximation of Maxwell and give approximate cross-sections for those processes involving a quark-antiquark pair plus four gluons. The generalization to larger numbers of gluons being straightforward and is given in ref.[6]. Detailed comparisons are made with the exact matrix elements and with the predictions of the effective-structure function approximation. When our results are combined with the approximate six-gluon cross-section of Maxwell they provide a powerful tool for analyzing the four-jet events of hadron colliders.

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2 Approximate Cross Sections

Our starting point is the exact non-zero tree level matrix element squared, to leading order in the number of colors, for the processes involving six gluons or quark-antiquark plus four gluons which maximally violates the conservation of helicity:

e.g. $g^+g^+ \to g^+g^+g^+g^+$, $q^+g^+ \to q^+g^+g^+g^+$ and $g^+g^- \to q^+\bar{q}^-g^+g^+$.

The matrix elements squared were given by Parke and Taylor[7],[8] for the purely gluonic process and by the authors[9] for the quark-antiquark plus four gluon process and can be simply given in terms of the elementary variables, $S_{ij} \equiv 2p_i \cdot p_j$. For the six gluon process the color sum for the matrix element squared for the sum of the maximally helicity violating amplitudes is given by

$$\sum |M_g^{viol}|^2 \;=\; g_s^8(Q^2)\; N^4(N^2-1)(\sum_{i<j} S_{ij}^4)(\sum_{perm'} \frac{1}{S_{12}S_{23}S_{34}S_{45}S_{56}S_{61}}) \qquad (1)$$

where $\sum_{perm'}$ is the sum over all non-cyclic permutations of the gluons $(1,2\ldots6)$. The similar expression for the quark-antiquark plus four gluons is

$$\begin{aligned}\sum |M_q^{viol}|^2 \;&=\; g_s^8(Q^2)\; 2N^3(N^2-1)\sum_i (S_{qi}^3 S_{\bar{q}i} + S_{qi}S_{\bar{q}i}^3)\\ &\quad\times\frac{1}{S_{\bar{q}q}}(\sum_{perm}\frac{1}{S_{q1}S_{12}S_{23}S_{34}S_{4\bar{q}}})\end{aligned} \qquad (2)$$

where $\sum_{perm}$ is now the sum over all permutations of the four gluons and $q, \bar{q}$ are the momenta of the quark and antiquark respectively.

For the six gluon process, Maxwell[4] has given us a method for including the contribution from the more complex helicity conserving amplitudes. His approximation is to multiply the matrix element squared for the helicity violating processes by a factor, χ_{gg}^g, such that the product has the Altarelli-Parisi[10] residue for the collinear pole of the pair of gluons with the smallest $|S_{ij}|$. Maxwell refers to this procedure as *infrared reduction.* For the purely gluonic process, the multiplication factor is

$$\chi_{gg}^g \;=\; \frac{(1+R)\;(1+z^4+(1-z)^4)}{(R+z^4+(1-z)^4)} \qquad (3)$$

where R and z are determined by the pair of gluons (α,β) which have the minimum $|S_{ij}|$ in the following way[3]

$$z = \frac{p_\alpha^0}{P^0} \quad , \quad P \equiv p_\alpha + p_\beta,$$

[3]This prescription is clearly not Lorentz invariant, but the violation of Lorentz invariance is an effect of order $s_{ij}/p_i^0 p_j^0$, which can be neglected consistently within the approximation.

$$R = \frac{\sum_{i<j,\neq\alpha,\beta} S_{ij}^4}{\sum_{i\neq\alpha,\beta} S_{iP}^4}. \tag{4}$$

In analogy with Maxwell's method one can show that the infrared reduction procedure applies to processes with a quark-antiquark pair plus gluons. Here the multiplication factor depends on the type of particles which make up the minimum $|S_{ij}|$. For the case where the particles with the minimum $|S_{ij}|$ are both gluons,

$$\chi_{gg}^q = \frac{(1+R)\,(1+z^4+(1-z)^4)}{(R+z^4+(1-z)^4)} \tag{5}$$

as before, but with

$$z = \frac{p_\alpha^0}{P^0} \quad , \quad P \equiv p_\alpha + p_\beta,$$

$$R = \frac{\sum_{i\neq\alpha,\beta}(S_{qi}^3 S_{\bar{q}i} + S_{qi} S_{\bar{q}i}^3)}{(S_{qP}^3 S_{\bar{q}P} + S_{qP} S_{\bar{q}P}^3)}. \tag{6}$$

If the pair with the minimum dot product is a quark and a gluon then

$$\chi_{qg}^q = \frac{(1+R)\,(1+z_q^2)}{(1+Rz_q^2)} \tag{7}$$

where

$$z_q = \frac{q^0}{Q^0} \quad , \quad Q \equiv p_\alpha + q,$$

$$R = \frac{\sum_{i\neq\alpha} S_{Qi}^3 S_{\bar{q}i}}{\sum_{i\neq\alpha} S_{Qi} S_{\bar{q}i}^3}. \tag{8}$$

The result for an antiquark-gluon pair is the same as the above quark-gluon pair but with each fermion momentum replaced by the appropriate anti-fermion momentum.

For the situation in which the minimum $|S_{ij}|$ pair is made up of a quark and an antiquark the multiplication factor is

$$\chi_{q\bar{q}}^q = (1+R) \tag{9}$$

where

$$G \equiv q + \bar{q},$$

$$R = \frac{\sum_{i<j} S_{ij}^4}{\sum_i S_{Gi}^4}. \tag{10}$$

Thus our approximation is equal to $\chi \cdot \sum |M^{viol}|^2$ times a weight factor which averages the incoming colors and helicities and also provides the appropriate statistical factor for identical particles. All of these results can be generalized to processes with more than four partons in the final state by expressing the approximate cross-section as a product of the maximally helicity-violating cross-section times more χ factors, two for a seven-parton process, three for an eight-parton process and so on.

3 The Comparison

To compare our results with the exact matrix elements squared we have looked at the processes $gg \to gggg$, $qg \to qggg$, $\bar{q}g \to \bar{q}ggg$ and $gg \to \bar{q}qgg$ in a proton - antiproton collider at $1.8TeV$, the Fermilab Tevatron. We omit the results for the $q\bar{q} \to gggg$ process because its rate is very small. Processes with two quark-pairs can be approximated in a similar way by using the simple helicity-violating matrix elements given in ref. [11], but their rate is totally negligible.

We have used a fixed set of structure functions throughout, Duke and Owens[12] ($\Lambda = 200MeV$), and the cuts on the partonic jets for the transverse momentum,P_T, pseudo-rapidity, y, and separation of the jets, ΔR, are as follows:

$$\begin{aligned} P_T &> 25GeV \\ |y| &< 3.5 \\ \Delta R \equiv \sqrt{\Delta\phi^2 + \Delta y^2} &> 0.8. \end{aligned} \tag{11}$$

We choose the Q^2 scale for the QCD evolution to be the average p_T of the event: $Q^2 = (\sum p_T/4)^2$. For both the exact and the approximate matrix elements we have plotted three differential cross sections, $\frac{d\sigma}{dP_T}$ versus P_T, $\frac{d\sigma}{dcos\theta_{23}}$ versus $cos\theta_{23}$, and $\frac{d\sigma}{dP_{out}}$ versus P_{out} in Figures 1 through 3. P_T is the transverse momentum of each jet. $P_{out} \equiv \frac{1}{2}\sum|p^i_{out}|$ with p^i_{out} the momentum of the i-th jet perpendicular to the plane given by the beam and the jet of largest p_T. The angle θ_{23} is the angle between the second and third highest energy jets in the center of mass of the incoming partons. For each differential cross-section there are four plots each appropriate for proton anti-proton collisions with five flavors of light quarks:

(a) for the purely gluonic process,

(b) for all processes with quark (antiquark) gluon to quark (antiquark) plus three gluons,

(c) for the process gluon gluon to quark antiquark plus two gluons and

(d) for the sum of these three.

The total rates for these processes are summarized in Table I.

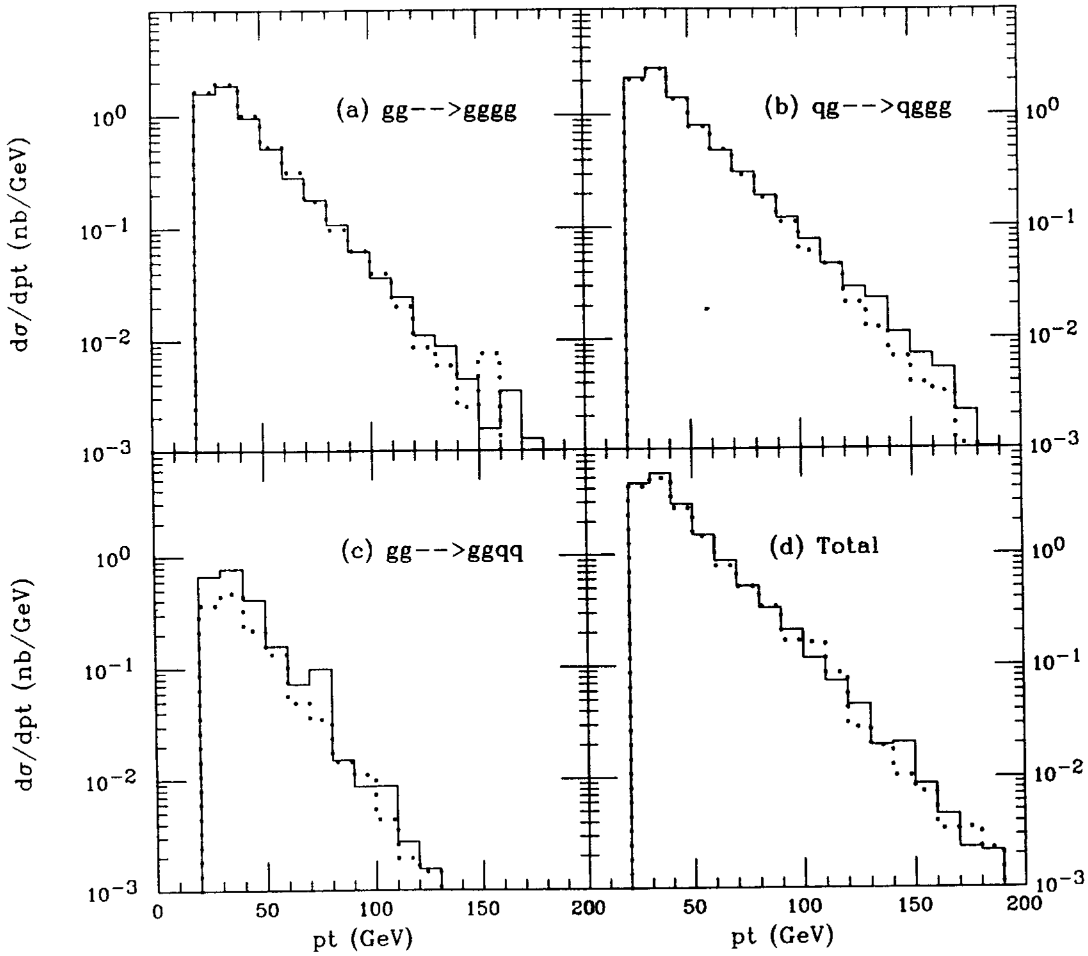

Figure 1(a)-(d): The differential cross sections $d\sigma/dP_T$ versus P_T for the labelled processes of four jet production at the Tevatron with the cuts given in eqn. (11). The solid line is the approximation and the dotted line the exact result.

Table I: Cross Sections for Four Jet Production at the Tevatron.

Process	Exact Cross Section nanobarns	Approximate Cross Section nanobarns
(a) $gg \to gggg$	14	15
(b) $qg \to qggg$	21	20
(c) $gg \to q\bar{q}gg$	3	6
(d) Total: Four Jets	38	41

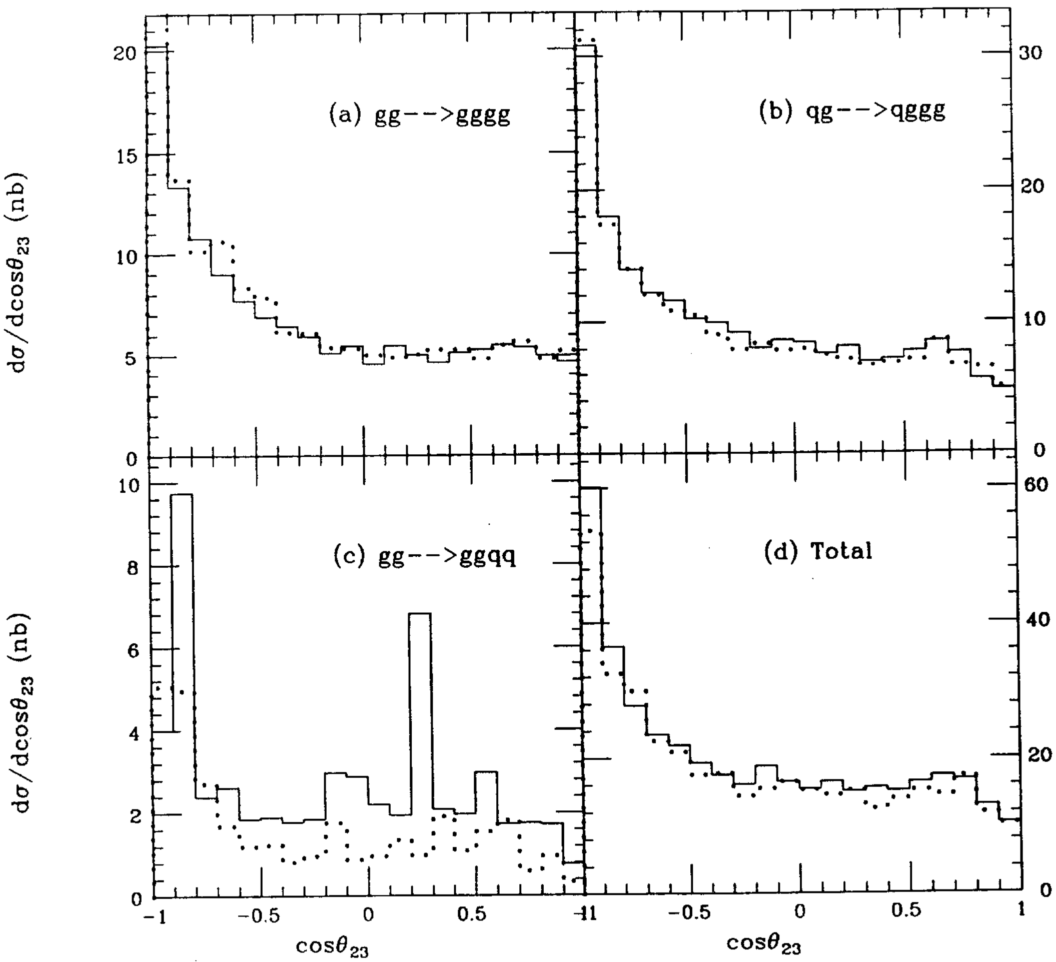

Figure 2(a)-(d): The differential cross sections $d\sigma/dcos\theta_{23}$ versus $cos\theta_{23}$ for the labelled processes of four jet production at the Tevatron with the cuts given in eqn. (11). The solid line is the approximation and the dotted line the exact result.

As is clear from Table I and from Figures (1) thru (3), the approximation to the purely gluonic process and to the processes with one quark in the initial state are extremely good, while the agreement between exact and approximated results for the process with a quark-antiquark pair in the final state is rather poor. Fortunately this last process has a small cross-section, and the induced error on the full cross-section is marginal.

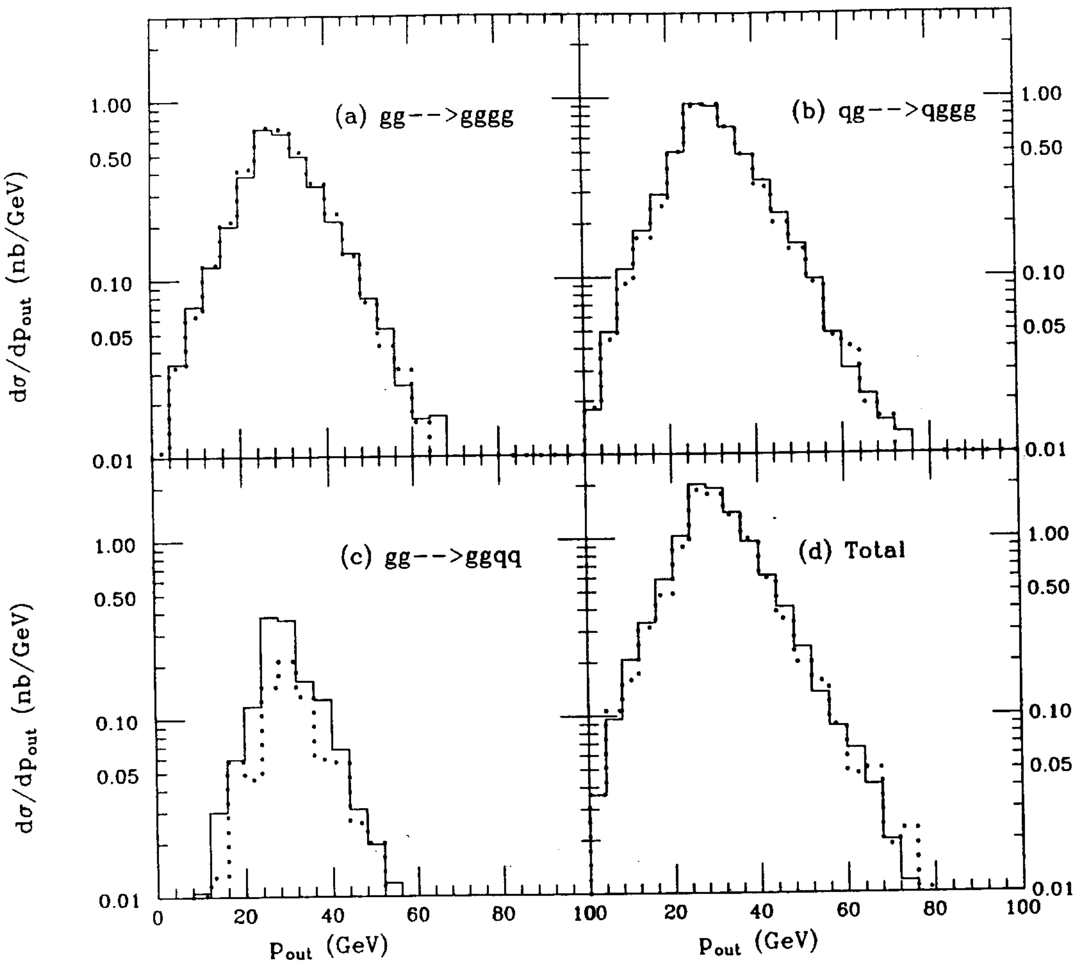

Figure 3(a)-(d): The differential cross sections $d\sigma/dP_{out}$ versus P_{out} for the labelled processes of four jet production at the Tevatron with the cuts given in eqn. (11). The solid line is the approximation and the dotted line the exact result.

The main reason underlying the accuracy of these approximations is the dominance of the helicity violating amplitudes over the helicity conserving ones. This in fact guarantees the stability of the infrared reduction when extrapolated from the collinear limit $s_{ij} \to 0$ to the observable kinematical configurations in which $s_{ij} \neq 0$. This dominance holds for the $gg \to gggg$ and $qg \to qggg$ processes. When integrating over phase space the helicity conserving amplitudes contribute in average to 20-30% of the full amplitude. Even a 30% uncertainty in estimating them (uncertainty coming from the extrapolation of the infrared reduction) would give rise to an error no larger than 10% on the full amplitude.

4 The Effective Structure Function Approximation

We now compare our approximation of the quark cross-section to the effective-structure-function approximation. This approximation amounts to assuming that in most of the relevant phase-space the differential cross-sections for processes initiated by gg, by qg and by qq or $q\bar{q}$ stand in a constant ratio:

$$\mathrm{d}\sigma_{gg} \; : \; \mathrm{d}\sigma_{gq} \; : \; \mathrm{d}\sigma_{qq} \; = \; 1 \; : \; 4/9 \; : \; (4/9)^2. \tag{12}$$

In this way the total cross-section, weighted by the appropriate structure functions, reads:

$$\mathrm{d}\sigma_{tot} \; = \; F(x_1)F(x_2)\mathrm{d}\sigma_{gg}, \tag{13}$$

$$F(x) = g(x) + 4/9\,(q(x) + \bar{q}(x)), \tag{14}$$

$g(x)$ and $q(x)$ being the gluon and quark structure functions.

This approximation is extremely good in the case of two partons in the final state, but becomes less and less accurate when increasing the complexity of the final state. Phenomenological applications of this approximation for multi-jet physics were given by Kunszt and Stirling in ref. [13]. These authors, however, used a simplified version of the multi-gluon approximation. Namely, they choose for the n gluon process a constant value given by the ratio of the total number of non-zero helicity configurations with the number of helicity-violating configurations contributing to a multi-gluon process,

$$\chi^g_{KS} \; = \; \frac{(2^n - 2n - 2)}{n(n-1)}. \tag{15}$$

For $n = 6$ we have $\chi^g_{KS} = 5/3$.

We have compared the prediction for the $qg \to qggg$ process obtained through the Kunszt and Stirling (KS) approximation and through the Maxwell (M) approximation with the exact calculation, which in turn agrees with our approximation (MP) within numerical (Monte Carlo) errors.

$$\mathrm{d}\sigma^q_{MP} = \; \chi^q \mathrm{d}\sigma^{viol}_q \tag{16}$$

$$\mathrm{d}\sigma^q_{KS} = \; 4/9 \;\; \chi^g_{KS}\mathrm{d}\sigma^{viol}_g \tag{17}$$

$$\mathrm{d}\sigma^q_{M} = \; 4/9 \;\; \chi^g_{gg}\mathrm{d}\sigma^{viol}_g. \tag{18}$$

The resulting distributions are shown in figs. 4 and 5. The total rates are as follows:

$$\sigma^q_{exact} = 21nb, \quad \sigma^q_{MP} = 20nb, \quad \sigma^q_{KS} = 30nb, \quad \sigma^q_{M} = 24nb. \tag{19}$$

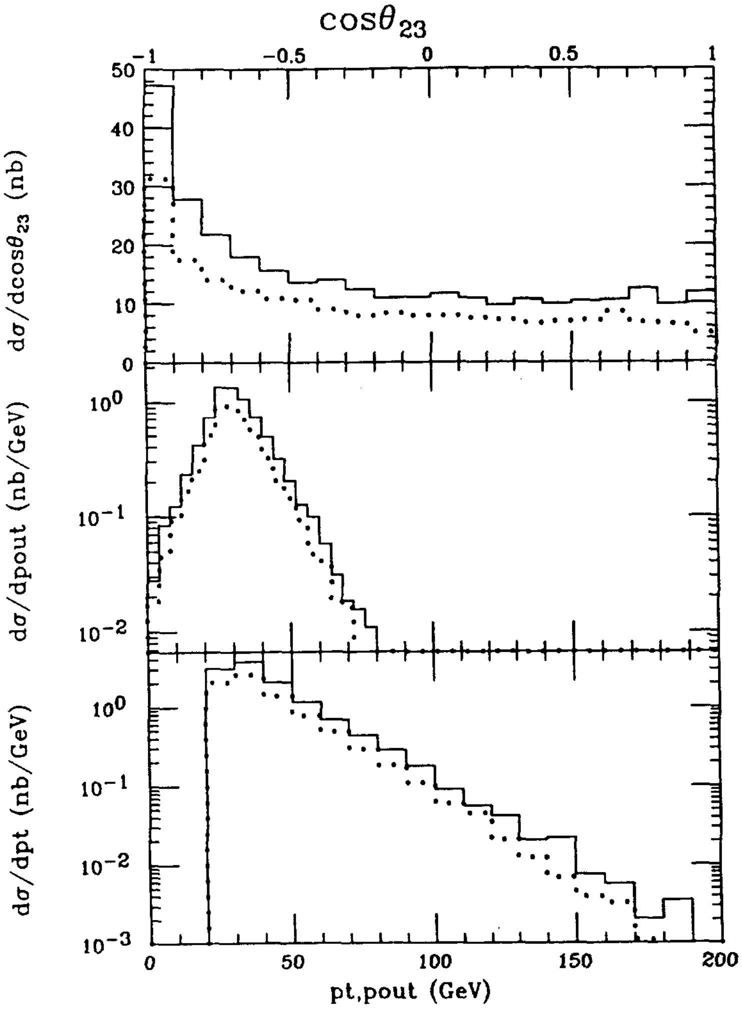

Figure 4: Comparison of the differential cross sections for the subprocess $qg \to qggg$ of our approximation (dots) versus the approximation of Kunszt and Stirling together with the use of the effective structure function approximation (solid).

From these results we conclude that the effective-structure-function approximation tends to overestimate the contribution of quark-initiated processes. This suggests that for a large number of partons the purely gluonic matrix elements dominates over the matrix elements with quarks (this is not necessarily true of the rates, because of the effect of the structure functions). However the mismatch between the exact result (or our approximation) and the result of the effective-structure-function approximation is certainly compatible with the *intrinsic* uncertainty associated with these calculations, due to the absence of higher order corrections, uncertainty in the choice of α_s, of Q^2 and of structure functions.

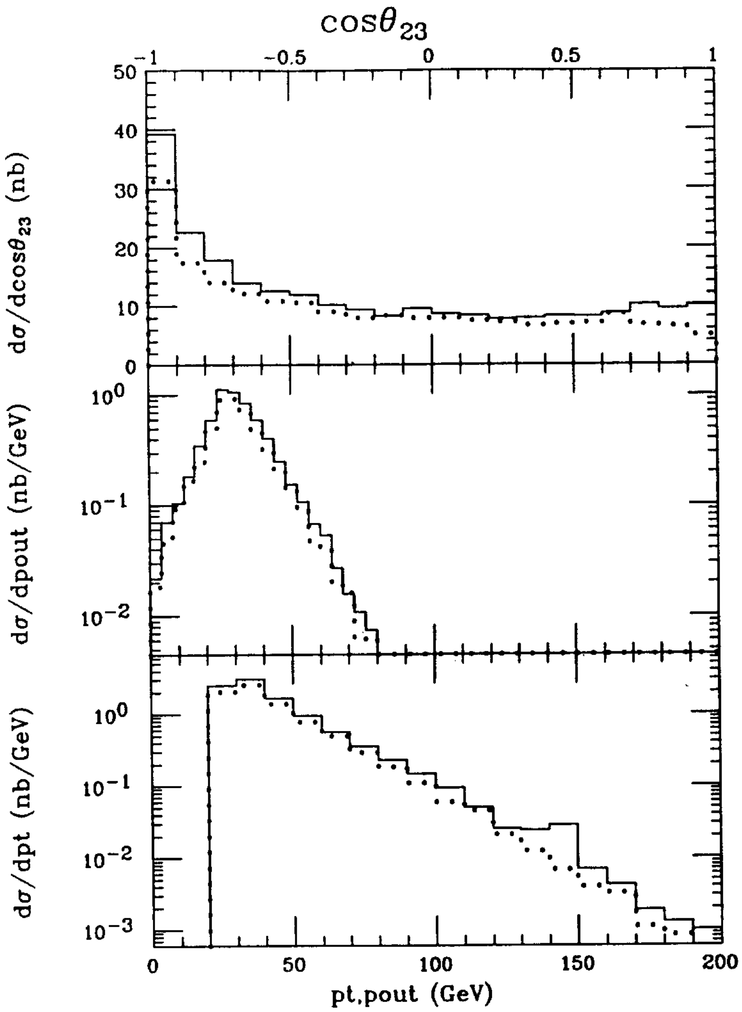

Figure 5: Comparison of the differential cross sections for the subprocess $qg \rightarrow qggg$ of our approximation (dots) versus the approximation of Maxwell together with the use of the effective structure function approximation (solid).

5 Conclusions

In conclusion, we have presented an approximation procedure to describe multi-jet QCD processes. Our prescription completes Maxwell's work on multi-gluon processes by generalizing it to processes involving quarks as well. The calculation of four-jet production in $p\bar{p}$ collisions at 1.8 TeV shows excellent agreement between the exact results and our approximation. The agreement holds for both total rates and differential distributions. This is a net improvement over calculations based on the effective-structure-function approximation, with which we have compared our results. Qualitative arguments suggest that this agreement should persist for higher order processes.

On completion of this manuscript we became aware of a preprint by C. Maxwell[14] which contains similar results to this paper.

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